

Game Theory

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1 Introduction to Game Theory and Examples

- 1.1.- Aim of Game Theory
- 1.2.- Decision Theory (one agent)
- 1.3.- Decision Theory (two players): Game Theory
- 1.4.- History of Game Theory
- 1.5.- Non-cooperative Games *versus* Cooperative Games
- 1.6.- Examples

REFERENCES

- Aumann, R. and M. Maschler. “Game Theoretic Analysis of a Bankruptcy Problem from the Talmud,” *Journal of Economic Theory* 36, 1985.
- Bertrand, J. “Théorie Mathématique de la Richesse Sociale,” *Journal des Savants* 67, 1883.
- Cournot, A.A. *Recherches sur les principes mathématiques de la théorie des richesses*. Paris: L. Hachette, 1838.
- Edgeworth, F.Y. *Papers Relating to Political Economy* (3 volumes). London: Royal Economic Society, 1925.
- Gale, D. “A Curious Nim-type Game,” *American Mathematical Monthly* 81, 1974.
- Hotelling, H. “Stability in Competition,” *Economic Journal* 39, 1929.
- Moulin, H. *Axioms of Cooperative Decision Making* (1st ed.). Cambridge: Cambridge University Press. Econometric Society Monographs, 1988.
- Nasar, S. *A Beautiful Mind*. New York: Simon & Schuster, 1998.
- Nash, J. “Equilibrium Points in N -Person Games,” *Proceedings of the National Academy of Sciences* 36, 1950.
- Nash, J. “The Bargaining Problem,” *Econometrica* 18, 1950.
- von Neumann, J. “Zur Theorie der Gesellschaftsspiele,” *Mathematische Annalen* 100, 1928.
- von Neumann, J. and O. Morgenstern. *The Theory of Games and Economic Behavior*. Princeton: Princeton University Press, 1944.

Ruffin, R.J. “Cournot Oligopoly and Competitive Behavior,” *Review of Economic Studies* 38, 1971.

Schmeidler, D. “The Nucleolus of a Characteristic Function Game,” *SIAM Journal on Applied Mathematics* 17, 1969.

Shapley, L.S. “The Solutions of a Symmetric Market Game,” *Annals of Mathematical Studies* 40, 1959.

Shapley, L.S. “Some Topics in Two-person Games,” in *Advances in Game Theory*, editors: M. Dresher, J. Shapley, and A. Tucker. Princeton: Princeton University Press, 1964.

von Stackelberg, H.F. *Markform und Gleichgewicht*. Vienna: Julius Springer, 1934.

Zermelo, E. “Über eine Anwendungen der Mengenlehre auf die Theorie der Schachspiels,” *Proceedings of the International Fifth Congress of Mathematicians*. Cambridge: Cambridge University Press, 1913.

2 Games in Normal Form

2.1.- Definition and Examples

2.2.- Nash Equilibrium

2.3.- Interpretation of Nash Equilibrium

2.4.- The Mixed Extension

2.5.- Computing Nash Equilibria

2.6.- General Existence Theorem

2.7.- Two-person Zero-sum Games: The Minimax Theorem

2.8.- Fictitious Play

REFERENCES

Aumann, R. “Acceptable Points in General Cooperative n -person Games,” in *Contributions to the Theory of Games IV*. Princeton: Princeton University Press, 1959.

Bernheim, B., B. Peleg, and M. Whinston. “Coalition-proof Nash Equilibria I: Concepts,” *Journal of Economic Theory* 42, 1987.

Debreu, G. “A Social Equilibrium Existence Theorem,” *Proceedings of the National Academy of Sciences* 38, 1952.

Fan, K. “Fixed Point and Minimax Theorems in Locally Convex Topological Linear Spaces,” *Proceedings of the National Academy of Sciences* 38, 1952.

Glicksberg, I.L. “A Further Generalization of the Kakutani Fixed Point Theorem with Application to Nash Equilibrium Points,” *Proceedings of the National Academy of Sciences* 38, 1952.

Harsanyi, J. and R. Selten. *A General Theory of Equilibrium Selection in Games*. MIT Press, 1988.

Kakutani, S. “A Generalization of Brouwer’s Fixed Point Theorem,” *Duke Mathematical Journal* 8, 1941.

Krishna, V. “Learning in Games with Stochastic Complementarities,” mimeo, 1991.

Milgrom, P. and J. Roberts. “Adaptive and Sophisticated Learning in Normal Form Games,” *Games and Economic Behavior* 3, 1991.

Miyasawa, K. “On the Convergence of the Learning Process in a 2×2 Non Zero-sum Two Person Game,” Economic Research Program, Princeton University, research memorandum No. 33, 1961.

Monderer, D. and A. Sela. “A 2×2 Game without the Fictitious Play Property,” *Games and Economic Behavior* 14, 1996.

Monderer, D. and L.S. Shapley. “Fictitious Play Property for Games with Identical Interests,” *Journal of Economic Theory* 68, 1996.

Nash, J. “Equilibrium Points in n -Person Games,” *Proceedings of the National Academy of Sciences* 36, 1950.

Nash, J. “Non-cooperative Games,” *Annals of Mathematics* 54, 1951.

von Neumann, J. and O. Morgenstern. *The Theory of Games and Economic Behavior*. Princeton: Princeton University Press, 1944.

Robinson, J. “An Iterative Method of Solving a Game,” *Annals of Mathematics* 54, 1951.

Schelling, T. *The Strategy of Conflict*. Cambridge: Harvard University Press, 1960.

Shapley, L.S. “Some Topics in Two-person Games,” in *Advances in Game Theory*, editors: M. Dresher, J. Shapley, and A. Tucker. Princeton: Princeton University Press, 1964.

Tarski, A. “A Lattice-Theoretical Fixpoint Theorem and Its Applications,” *Pacific Journal of Mathematics* 5, 1955.

Vives, X. “Nash Equilibrium with Strategic Complementarities,” *Journal of Mathematical Economics* 19, 1990.

3 Games in Extensive Form

3.1.- Preliminaries

3.2.- Perfect Information

3.3.- Nash Equilibrium: Backwards Induction and Kuhn's Theorem

3.4.- Imperfect Information

REFERENCES

Kuhn, W. "Extensive Games and the Problem of Information," *Annals of Mathematical Study* 28, 1953.

Kuhn, W. and A. Tucker (editors). *Contributions to the Theory of Games II*. Princeton: Princeton University Press, 1953.

Zermelo, E. "Über eine Anwendungen der Mengenlehre auf die Theorie der Schachspiels," *Proceedings of the International Fifth Congress of Mathematicians*. Cambridge: Cambridge University Press, 1913.

4 Nash Equilibrium and Related Issues

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4.2.- Dominant Strategies

4.3.- Elimination of Dominated Strategies

4.4.- Subgame Perfect Equilibrium

4.5.- Trembling-hand Perfect Equilibrium

4.6.- Perfect Equilibrium in the Normal Form

4.7.- Perfect Equilibrium and Undominated Strategies

4.8.- Proper Equilibrium

4.9.- Correlated Equilibrium

REFERENCES

Aumann, R. "Subjectivity and Correlation in Randomized Strategies," *Journal of Mathematical Economics* 1, 1974.

Bernheim, B. "Rationalizable Strategic Behavior," *Econometrica* 53, 1984.

Calvo-Armengol, A. "The Set of Correlated Equilibria of 2×2 Games," mimeo, 2003. <http://selene.uab.es/acalvo/correlated.pdf>

Foster, D. and R. Vohra. "Calibrated Learning and Correlated Equilibrium," *Games and Economic Behavior* 21, 1997.

Fudenberg, D. and D. Levine. "Conditional Universal Consistency," *Games and Economic Behavior* 29, 1999.

Hart, S. and A. Mas-Colell. "A Simple Adaptive Procedure Leading to Correlated Equilibrium," *Econometrica* 68, 2000.

Hart, S. and A. Mas-Colell. "A General Class of Adaptive Strategies," *Journal of Economic Theory* 98, 2001.

Hillas, J. "On the Definition of the Strategic Stability of Equilibria," *Econometrica* 58, 1990.

Kholberg, E. and J.F. Mertens. "On the Strategic Stability of Equilibria," *Econometrica* 54, 1986.

Moulin, H. "Dominance Solvable Voting Schemes," *Econometrica* 47, 1979.

Moulin H. "On Strategy Proofness and Single-Peakedness," *Public Choice* 35, 1980.

Myerson, R. "Refinements of the Nash Equilibrium Concept," *International Journal of Game Theory* 7, 1978.

Pearce, D. "Rationalizable Strategic Behavior and the Problem of Perfection," *Econometrica* 52, 1984.

Rosenthal, R. "Games of Perfect Information, Predatory Pricing and the Chain-store Paradox," *Journal of Economic Theory* 25, 1981.

Selten, R. "Spieltheoretische Behandlung eines Oligopolmodells mit Nachfragertrgheit," *Zeitschrift für die gesamte Staatswissenschaft* 12, 1965.

Selten, R. "Reexamination of the Perfectness Concept for Equilibrium Points in Extensive Games," *International Journal of Game Theory* 4, 1975.

van Damme, E. *Stability and Perfection of Nash Equilibria*, Springer-Verlag, 1991.

5 Repeated Games

5.1.- Introduction

5.2.- Strategies

5.3.- Payoffs

5.4.- "Folk" Theorems

5.5.- Stochastic Games

REFERENCES

Aumann, R. "Survey of Repeated Games," in *Essays in Game Theory and Mathematical Economics in Honor of Oskar Morgenstern*, 1981.

Aumann, R. and L. Shapley. "Long Term Competition -A Game Theoretic Analysis," Mimeo, The Hebrew University, 1976.

Benoît, J.P. and V. Krishna. “Finitely Repeated Games,” *Econometrica* 53, 1985.

Benoît, J.P. and V. Krishna. “Nash Equilibria of Finitely Repeated Games,” *International Journal of Game Theory* 16, 1987.

Friedman, J. “A Non-cooperative Equilibrium for Supergames,” *The Review of Economic Studies* 38, 1971.

Fudenberg, D. and E. Maskin. “The Folk Theorem in Repeated Games with Discounting or with Incomplete Information,” *Econometrica* 54, 1986.

Lockwood, B. “The Folk Theorem in Stochastic Games with and without Discounting,” *Birkbeck College Discussion Paper in Economics* 18, 1990.

Massó, J. and A. Neme. “Equilibrium Payoffs of Dynamic Games,” *International Journal of Game Theory* 25, 1996.

Rubinstein, A. “Equilibrium in Supergames,” Mimeo, The Hebrew University, 1977.

Shapley, L. “Stochastic Games,” *Proceedings of the National Academy of Sciences* 39, 1953.

6 Games of Incomplete Information

6.1.- The Harsanyi Solution

6.2.- Bayesian-Nash Equilibrium

6.3.- Sequential Equilibrium

6.4.- Using Bayesian-Nash Equilibria to Justify Mixed Equilibria

6.5.- Signalling Games and Forward Induction

REFERENCES

Harsanyi, J. “Games with Incomplete Information Played by “Bayesian” Players,” *Management Science* 14, 1967-68.

Harsanyi, J. “Games with Randomly Distributed Payoffs: A New Rationale for Mixed-Strategy Equilibrium Points,” *International Journal of Game Theory* 2, 1973.

Kreps, D. and R. Wilson. “Sequential Equilibria,” *Econometrica* 50, 1982.

Kreps, D. and R. Wilson. “Reputation and Imperfect Information”. *Journal of Economic Theory* 27, 1982.

Mertens, J.F. and S. Zamir. “Formulation of Bayesian Analysis for Games with Incomplete Information,” *International Journal of Game Theory* 14, 1985.

van Damme, E. *Stability and Perfection of Nash Equilibria*, Springer-Verlag, 1991.

7 Bargaining Theory

- 7.1.- Bargaining Theory
- 7.2.- The Bargaining Problem
- 7.3.- The Nash Bargaining Solution
- 7.4.- The Kalai-Smorodinsky Solution
- 7.5.- Strategic Bargaining

REFERENCES

Binmore, K. “Nash Bargaining Theory II”. In *The Economics of Bargaining*, editors: K. Binmore and P. Dasgupta, 1987.

Kalai, E. and M. Smorodinsky. “Other Solutions to Nash’s Bargaining Problem,” *Econometrica* 45, 1975.

Nash, J. “The Bargaining Problem,” *Econometrica* 18, 1950.

Nash, J. “Two-Person Cooperative Games,” *Econometrica* 21, 1953.

Osborne, M.J. and A. Rubinstein. *Bargaining and Markets*. Academic Press, 1990.

Roth, A. “Individual Rationality and Nash’s Solution to the Bargaining Problem”. *Mathematics of Operations Research* 2, 1977.

Roth, A. *Axiomatic Models of Bargaining*. Springer-Verlag, 1979.

Rubinstein, A. “Perfect Equilibrium in a Bargaining Model,” *Econometrica* 50, 1982.

8 Cooperative Games

- 8.1.- Preliminaries.
- 8.2.- Core
- 8.3.- Stable Sets
- 8.4.- Bargaining Sets
- 8.5.- Shapley Value
- 8.6.- Nucleolus

REFERENCES

- Aumann, R. and M. Maschler. "The Bargaining Set for Cooperative Games," in *Advances in Game Theory*, editors: M. Dresher, J. Shapley, and A. Tucker. Princeton University Press, 1964.
- Bondareva, O.N. "Certain Applications of the Methods of Linear Programming to the Theory of Cooperative Games," *Problemy Kibernet* 10, 1963.
- Gillies, D. B. *Some Theorems on n -Person Games*. Ph.D. Thesis, Princeton University Press, 1953.
- Hurt, S. and A. Mas-Colell. "Potential, Value and Consistency," *Econometrica* 57, 1989.
- Mas-Colell, A. "An equivalence Theorem for a Bargaining Set," *Journal of Mathematical Economics* 18, 1989.
- Maschler, M. "The Bargaining Set, Kernel, and Nucleolus". In *Handbook of Game Theory with Economic Applications I*, editors: R. Aumann and S. Hart. North-Holland, 1992.
- Lucas, W.F. "The Proof That a Game May Not Have a Solution," *Transactions of the American Mathematical Society* (American Mathematical Society) 136, 1969.
- von Neumann, J. and O. Morgenstern. *The Theory of Games and Economic Behavior*. Princeton University Press, 1944.
- Schmeidler, D. "The Nucleolus of a Characteristic Function Game," *SIAM Journal on Applied Mathematics* 17, 1969.
- Shapley, L. "A Value for n -Person Games". In *Contributions to the Theory of Games II*, editors: H. Kuhn and A. Tucker. Princeton University Press, 1953.
- Shapley, L. "On Balanced Sets and Cores," *Naval Research Logistics Quarterly* 14, 1967.
- Shapley, L. "Utility Comparison and the Theory of Games". In *La Decision: Aggregation et Dynamique des Ordres de Preference*, Editions du Centre National de la Recherche Scientifique, 1969.
- Sobolev, A.I. "The characterization of optimality principles in cooperative games by functional equations," *Mathematical Methods in the Social Sciences* 6, 1975.
- Vohra, R. "An Existence Theorem for a Bargaining Set," *Journal of Mathematical Economics* 20, 1991.
- Young, H. P. "Monotonic Solutions of Cooperative Games," *International Journal of Game Theory* 14, 1985.
- Zhou, L. "A New Bargaining Set of an N -Person Game and Endogenous Coalition Formation," *Games and Economic Behavior* 6, 1994.

Text Books

- Binmore, K. *A Primer in Game Theory*. D. C. Health and Company, 1992.
- van Damme, E. *Stability and Perfection of Nash Equilibria*, Springer-Verlag, 1991.
- Driessen, T. *Cooperative Games, Solutions and Applications*. Kluwer Academic Publishers, 1988.
- Friedman, J. *Game Theory with Applications to Economics* (second edition). Oxford University Press, 1991.
- Fudenberg, D. and J. Tirole. *Game Theory*. MIT Press, 1991.
- Gibbons, R. *A Primer in Game Theory*. Harvester Wheatsheal, 1992.
- Harsanyi, J. and R. Selten. *A General Theory of Equilibrium Selection in Games*. MIT Press, 1988.
- Kreps, D. *Game Theory and Economic Modeling*. Clarendon Press, 1990.
- Luce, R., and H. Raiffa. *Games and Decisions*. Wiley, 1957.
- Mas-Colell, A., M. Whinston, and J. Green. *Microeconomic Theory*. Oxford University Press, 1995.
- Maschler, M., Solan, E. and S. Zamir. *Game Theory*. Cambridge University Press, 2013.
- Moulin, H. *Game Theory for the Social Sciences* (second edition). New York University Press, 1986.
- Moulin, H. *Axioms of Cooperative Decision Making*. Cambridge University Press (Econometric Society Monographs), 1988.
- Myerson, R. *Game Theory: Analysis of Conflict*. Harvard University Press, 1991.
- von Neumann, J. and O. Morgenstern. *The Theory of Games and Economic Behavior*. Princeton University Press, 1944.
- Osborne, M.J. *An Introduction to Game Theory*. Oxford University Press, 2004.
- Osborne, M.J. and A. Rubinstein. *A Course in Game Theory*. MIT Press, 1994.
- Owen, G. *Game Theory* (second edition). Academic Press, 1982.
- Schelling, T. *The Strategy of Conflict*. Harvard University Press, 1960.
- Shubik, M. *Game Theory in the Social Sciences*. MIT Press, 1984.
- Vega-Redondo, F. *Economics and the Theory of Games*. Cambridge University Press, 2003.